How to Teach the Critical Difficulties of Arithmetic

to accompany the

CALIFORNIA STATE SERIES ADVANCED ARITHMETIC

For Normal Students and Experienced Teachers

By MARY A. WARD

Supervisor of the Teaching of Arithmetic

15484

CALIFORNIA
STATE PRINTING OFFICE
1915

TABLE OF CONTENTS.

HOW TO TEACH THE CRITICAL DIFFICULTIES OF ARITHMETIC.

INTRODUCTION.

The helps and suggestions given in this bulletin for teaching each of the topics of the State Series Advanced Arithmetic may be applied to the work of any arithmetic text. Teachers will find the miscellaneous questions and tests given at the end of each topic useful in many ways. They may be used as thorough reviews at the beginning of a new term or as tests for new pupils who enter late in the term. Such tests should help to determine the points in which a pupil's knowledge is deficient. The classification of the problems of pages 18 through 165 of the State text will be of much service to the teacher who is using this portion of the textbook.

High school students desiring to enter the Normal School, will find that the bulletin outlines the scope and type of the arithmetic work with which students must be familiar before they can be entrusted with the teaching of this subject in the classes of the training school. Students will be expected to be familiar with the tests, suggestions and methods of work given in this book.

Besides the subjects treated in this outline, normal students are expected to be familiar with the following:

1. The system of addition combinations as given in chapter three of the State primary text.
2. The method of finding the quotient figures in long division. (Advanced State Text, pages 63–67.)
3. Additive method of subtraction. (Advanced Text, pages 21–22.)

MARY A. WARD.

State Normal School, San Francisco.
February, 1915.

HOW TO TEACH THE CRITICAL DIFFICULTIES OF ARITHMETIC.

COMMON FRACTIONS.

The following sections summarize the various types of problems which should be taught in the addition, subtraction, multiplication and division of fractions. The suggestions given here for the teaching of fractions are intended to be used in connection with the indicated sections of the California State Series Advanced Arithmetic, but may be used in connection with the fraction work of any arithmetic text. Normal students will be required to work problems similar to the types given here and to use the form and methods of work indicated in this outline. In order to follow the references, students should if possible provide themselves with a copy of the California State Series Advanced Arithmetic. Students should be especially familiar with the work of Sections 26 and 27.

1. Fractions:

1. Teach pupils that a piece and a fraction of an object are one and the same.

2. That the number of equal pieces into which the whole quantity is divided determines the name of the piece or fraction. Ask—"If a line is divided into 4 equal parts, what is each part called? If divided into 2 equal parts? 5 equal parts? etc." (Text page 90.)

3. Ask—"How many fourths, thirds, fifths, etc. are there in one whole?"

4. Teach pupils the names of the parts of a fraction—numerator and denominator. (Text page 93.)

5. Suggest that the first letter of the word down is **d**. The name of the term which is down or below the line, **denominator**, also begins with a **d**.

6. Have pupils show $\frac{1}{4}$, $\frac{3}{5}$, $\frac{2}{3}$, $\frac{7}{8}$, etc., of lines, rectangles, circles. Be careful to see that pupils point not to the line of division, but to the portion of the line or surface when showing $\frac{1}{4}$, $\frac{2}{3}$, $\frac{7}{8}$, etc., of a line, rectangle, or circle. (Text page 93, numbers 6–7.)

7. Teach pupils to distinguish between proper and improper fractions and mixed numbers. Do not teach definitions of these terms. (Text page 94.)

8. Have pupils classify each of the following as proper fractions, improper fractions or as mixed numbers: $1\frac{2}{3}$, $\frac{7}{7}$, $\frac{6}{7}$, $\frac{8}{9}$, $\frac{10}{5}$, $1\frac{3}{4}$, $\frac{11}{3}$. Pupils often fail to regard $\frac{7}{7}$, $\frac{8}{8}$, etc., as improper fractions.

Write a miscellaneous list of proper, improper fractions and mixed numbers on the board. Have pupils tell whether each is I (improper), M (mixed number) or P (proper). This form of drill may be either oral or written.

2. Changing Improper Fractions to Mixed Numbers.

Before beginning the addition of fractions of like denominators, it is necessary that pupils know how to change improper fractions to mixed numbers. **There is no necessity for teaching pupils to change mixed numbers to improper fractions at this time.** The application of the drill upon changing mixed numbers to improper fractions can not be made until pupils are ready for work in the multiplication of fractions. For this reason the work of Sections 119–120 of text should be delayed until pupils are ready to begin the multiplication of fractions.

3. Change $\frac{19}{3}$ to a Mixed Number: (Text page 96.)

a. Teach the pupils to read $\frac{19}{3}$ as 19 divided by 3. Emphasize the fact that the horizontal line is the sign of division. Pupils should read $\frac{19}{3}$ in two ways—as 19 thirds or as 19 divided by 3.

b. Ask: What is the answer when 19 is divided by 3? Nineteen thirds equal what? Proceed in the same manner with other improper fractions. Do not teach by rule the changing of improper fractions to mixed numbers.

c. When pupils can readily change improper fractions to mixed numbers, show the reason the numerator is divided by the denominator.

Ask: How many thirds are there in one whole? How many whole numbers are there in $\frac{3}{3}$, $\frac{6}{3}$, $\frac{9}{3}$, $\frac{12}{3}$, $\frac{15}{3}$, etc., $\frac{4}{3}$, $\frac{5}{3}$, $\frac{7}{3}$, $\frac{11}{3}$, $\frac{13}{3}$, $\frac{19}{3}$?

d. Require pupils to change mentally all small improper fractions to mixed numbers.

4. Reduction to Lower Terms.

Much time is wasted in attempting to teach pupils to find the greatest common divisor of two or more numbers. That the reduction of fractions to lower terms may be taught without the use of such terms is shown by the following suggestions (Text 87–89):

a. Teach that the even numbers are 0, 2, 4, 6, 8.

b. Ask: Which of the following end in even numbers 234, 40, 43, 27, 22, 35, 19, 36?

c. Teach pupils to recognize which of the following fractions have numerators and denominators that end in even numbers.

$\frac{24}{28}$, $\frac{16}{21}$, $\frac{20}{24}$, $\frac{32}{38}$, $\frac{37}{48}$ etc.

d. **Teach that when both the numerator and the denominator of a fraction end in even numbers, the fraction may be reduced by dividing both numerator and denominator by 2.**

5. Reduction by 5: Teach that when the numerator and the denominator of a fraction end in **5**, or **0 and 5**, the fraction may be reduced by dividing both numerator and denominator of the fraction by 5. Thus, $\frac{255}{400} = \frac{51}{80}$.

6. Reduction by 10: Teach that when both the numerator and the denominator of a fraction end in **0**, the fraction may be reduced by dividing both the numerator and the denominator of the fraction by 10. Thus, $\frac{60}{70} = \frac{6}{7}$.

7. **Reduction by 3:** Reduce $\frac{12}{39}$.

a. In the numerator, 12, add together the 1 and 2. The answer is 3. May 3 be divided by 3?

b. Add together the 3 and 9 in the denominator, 39. The answer is 12.

c. May 12 be divided by 3?

d. May the sum of the numbers in the numerator of $\frac{12}{39}$ be divided by 3?

e. May the sum of the numbers in the denominator of $\frac{12}{39}$ be divided by 3?

f. **When the sum of the numbers in the numerator and the sum of the numbers in the denominator of a fraction are divisible by 3, the original fraction is divisible by 3, thus,** $\frac{12}{39} = \frac{4}{13}$.

g. Reduce each of the following fractions by dividing both the numerator and the denominator of the fraction by 3:

$$\frac{231}{1728}, \frac{48}{63}, \frac{51}{72}, \frac{45}{72}.$$

h. Give much drill upon the reduction of small fractions such as $\frac{3}{6}$, $\frac{4}{6}$, $\frac{2}{8}$, $\frac{4}{12}$, $\frac{2}{6}$, etc. Pupils should be able to reduce such types mentally without any hesitation.

8. **Summary of Types in Reduction to Lower Terms:** (Text pages 101, 104, 105.)

a. $\frac{234}{632}$, $\frac{182}{642}$. Divide by 2 as the numerator and the denominator end in even numbers.

b. $\frac{685}{1050}$, $\frac{205}{135}$. Divide by 5 as the numerator and the denominator end in 5, or 0 and 5.

c. $\frac{230}{450}$, $\frac{100}{120}$. Divide by 10 as the numerator and the denominator end in 0.

d. $\frac{231}{432}$, $\frac{102}{201}$. Divide by 3 as the sum of digits of numerator and the sum of digits of denominator are both divisible by 3.

9. **Difficulties:**

1. Besides the above types, pupils often fail to see the number by which fractions, such as the following, may be reduced: $\frac{17}{34}$, $\frac{13}{39}$, $\frac{11}{22}$, $\frac{14}{28}$, $\frac{19}{38}$.

2. Pupils often reduce a fraction once but fail to reduce it to its lowest terms, thus, $\frac{24}{36} = \frac{4}{6} = \frac{2}{3}$.

3. Train pupils to divide both numerator and denominator of fractions by **largest number** possible.

10. **Types in Addition of Fractions of Like Denominations:**

For drill material see text page 97.

a.	*b.*	*c.*	*d.*	*e.*
$\frac{1}{8}$	$\frac{3}{8}$	$\frac{2}{8}$	$24\frac{1}{8}$	$6\frac{3}{8}$
$\frac{2}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{5}{8}$
$\frac{2}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$4\frac{6}{8}$	$4\frac{6}{8}$
$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	9	25
$\frac{1}{8}$	$\frac{3}{8}$	$\frac{7}{8}$	$4\frac{5}{8}$	$14\frac{2}{8}$
$\frac{7}{8}$	$1\frac{1}{2}$	2	$42\frac{7}{8}$	51

Teach each of the above types in the given order. Do not pass from type *a* to *b* until pupils understand type *a* perfectly. Only a few problems are needed to teach the principles of each type. Follow closely the suggestions given under the following sections:

1. Notice that in type *a* the answer is a fraction only. Construct problems to fulfill this condition.

2. In type *b* the answer $\frac{1}{8}^2$ is to be changed **mentally** to the mixed number, $1\frac{1}{8}$ or $1\frac{1}{2}$. Nothing is put upon the paper except the answer in its simplest form. Notice the position of the $\frac{1}{2}$ in the answer. Where is the whole number, 2, placed? From the beginning of work with this type, insist upon pupils making all reductions mentally.

3. In type *c* pupils must be shown where to place the whole number **2.**

4. In type *d*, note the irregularity of the column. All changing and carrying is done mentally.

5. In type *e*, note the irregularity of the column and the position of the answer.

6. While teaching addition of fractions of like denominators, continue drill work upon the reduction of fractions to lower terms and the changing of improper fractions to mixed numbers.

11. Language Forms for Addition of Fractions:

Use the following language forms in dictating exercises in addition:

a. Find the sum of $4\frac{2}{3}$, $6\frac{1}{3}$, etc.

b. Add the following: $9\frac{2}{3}$, $6\frac{1}{3}$, etc.

c. Add: $6\frac{2}{3}$, $8\frac{1}{3}$, etc.

12. Addition of Fractions of Unlike Denominators:

Before introducing work in the addition of fractions of unlike denominators, pupils should be able — *a.* to change fractions to higher terms; *b.* to find the smallest common denominator for several unlike fractions.

13. Changing Fractions to Higher Terms: (Text pages 102–103.)

a. Teach pupils to change all possible fractions to 6ths, 8ths, 12ths, 18ths, 20ths, 24ths, 30ths, 36ths, 48ths, 60ths.

b. For purposes of illustrating the changing of fractions to higher terms, use fractions such as $\frac{3}{4}$, $\frac{5}{8}$, $\frac{2}{3}$ rather than $\frac{1}{4}$, $\frac{1}{8}$, or $\frac{1}{3}$. Since the numerators of the latter are 1, they are exceptions to the general rule.

14. The Smallest Common Denominator: (Text page 112, col. 3.)

a. Teach pupils that 1, 2, 3, 5, 7 are **prime** numbers. That is, such numbers are divisible only by themselves and one.

b. Teach pupils to find the smallest common denominator for unlike fractions by the following method:

Find the smallest common denominator for $\frac{3}{4}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{1}{12}$, $\frac{3}{7}$, $\frac{1}{2}$.

c. Write the denominators in a horizontal line. Thus,

$$\begin{array}{r} 2\,)\ 4 - 3 - 6 - 8 - 12 - 7 - 2 \\ 2\,)\ \quad\quad 4 - \ 6 - 7 \\ \quad\quad\quad\quad 2 - \ 3 - 7 \end{array}$$

$$2 \times 2 \times 2 \times 3 \times 7 = 168.$$

2—15484

d. Have pupils cross out all the small denominators which are contained an even number of times in the larger denominators. Since 4—3—6—2 are contained in the larger number, 12, they are crossed out.

e. Have pupils divide the remaining numbers by the smallest prime number which is contained in at least **two** of the numbers that are left. Continue as shown in the model. Emphasize the fact that the divisor must be contained in at least **two** of the numbers to be divided.

f. Pupils should be perfectly familiar with the method of finding the smallest common denominator **before beginning the addition of fractions of unlike denominators.** The above method of finding the common denominator will be more successful with the ordinary pupil than the inspection method given on pages 106 and 107 of the Advanced Text.

15. **Difficulties in Teaching Pupils to Find the Common Denominator:**

a. Pupils sometimes cross out both the large and small numbers. Teach that the larger numbers must be left to take care of the smaller numbers.

b. Pupils often fail to cross out small numbers when it is possible to do so in the second or third line of the work thus: Find the smallest common denominator for $\frac{3}{4}$, $\frac{5}{12}$, $\frac{1}{8}$, $\frac{4}{9}$.

$$\begin{array}{rl} 2) & 4 - 12 - 8 - 9 \\ 2) & \quad 6 - 4 - 9 \\ & \quad 3 - 2 - 9 \end{array}$$

$$2 \times 2 \times 2 \times 9 = 72.$$

c. Pupils often **waste time** dividing by a number which is contained in only **one** of the denominators. In the above model, pupils might divide the last line by 2 and then the next line by 9, which would be a great waste of time. Insist upon pupils dividing by the smallest number which is contained in at least **two** of the denominators.

d. In teaching the above work, always dictate the unlike fractions which are to be added. Do this so that pupils may see that they are learning to find the smallest common denominator of several numbers solely for the purpose of adding unlike fractions.

e. Be certain that the fractions which you dictate for the above work have denominators which illustrate the several steps of finding the smallest common denominator.

16. **Forms for Addition of Fractions of Unlike Denominators.**

All that remains to be taught in the addition of fractions of unlike denominators is the form of the work.

a.	*b.*
$6\frac{3}{4} = 6\frac{18}{24}$	$6\frac{3}{4}$ | $\frac{18}{24}$
$9\frac{2}{3} = 9\frac{16}{24}$	$9\frac{2}{3}$ | $\frac{16}{24}$
$8\frac{1}{2} = 8\frac{12}{24}$	$8\frac{1}{2}$ | $\frac{12}{24}$
$4\frac{3}{8} = 4\frac{9}{24}$	$4\frac{3}{8}$ | $\frac{9}{24}$
$29\frac{7}{24}$	29 | $\frac{7}{24}$

$4 - 3 - 2 - 8 = 24$ S. C. D.

a. Pupils who are learning the addition of fractions should use the form of model *a*.

b. Model *b* is shorter and may be used after the pupils have passed the work of multiplication of fractions, or sooner if the teacher feels that the pupils know what they are doing.

c. The work of finding the new denominator should always appear on the pupil's paper. In this way a pupil's knowledge of this principle may be tested.

d. Notice that the answer to the addition problem of the above model is written on the paper only in its simplest form. Train pupils to do as much as possible of the reduction mentally.

e. **Train pupils to reduce fractions of the given problem to lowest terms before attempting to find the common denominator of the fractions.** In this way much useless work is saved.

Find the sum of $\frac{4}{12}$, $\frac{8}{16}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{8}$, $\frac{4}{16}$. First reduce the fractions to lower terms. The problem then becomes: Find the sum of $\frac{1}{3}$, $\frac{1}{5}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{4}$, $\frac{1}{4}$.

17. Difficulties Found in the Teaching of Addition of Fractions.

a. The changing of fractions to higher terms.

b. Finding the smallest common denominator for several unlike fractions.

c. Changing mentally improper fractions to mixed numbers and reducing fractions to lower terms.

d. Failure of pupils to use the correct form when adding fractions of unlike denominators. Teach pupils why the following form is incorrect: $2\frac{3}{4} = \frac{9}{12}$.

e. Be careful to see that the problems you use for drill work illustrate the principle upon which you are drilling. In teaching the method of finding smallest common denominator, do not use a problem such as $\frac{2}{3} + \frac{1}{5} + \frac{3}{7} + \frac{1}{8}$.

f. Do not introduce the subtraction of fractions until pupils have a thorough knowledge of each of the above difficulties.

SUBTRACTION OF FRACTIONS.

18. Types in Subtraction of Fractions Which Involve No Carrying:

For review work after types *a*, *b*, *c*, and *d* have been taught, use the material on pages 98–99 of text.

Type a. Larger Fraction in Minuend:

$$\begin{array}{r} 295\frac{3}{4} \\ -\ 27\frac{1}{4} \\ \hline \end{array} \qquad \begin{array}{r} \frac{3}{4} \\ -\frac{1}{4} \\ \hline \end{array}$$

Type b. Fractions in Minuend Only:

$$\begin{array}{r} 249\frac{3}{4} \\ -\ 18 \\ \hline \end{array}$$

Type c. Fraction Only Subtracted From Mixed Number:

$$\begin{array}{r} 832\frac{3}{4} \\ -\ \frac{3}{4} \\ \hline \end{array}$$

Type d. Answer a Whole Number Only:

$$\begin{array}{r} 23\frac{4}{9} \\ -\ 18\frac{4}{9} \\ \hline \end{array} \qquad \begin{array}{r} 23\frac{4}{9} \\ -\ \frac{4}{9} \\ \hline \end{array}$$

Suggestions:

1. Teach each of the above types one at a time. Do not attempt to make pupils conscious of which types they are learning.

2. Insist upon pupils making all reductions **mentally.**

3. Teach pupils **to prove their answers** to all problems in the subtraction of fractions.

4. Be certain that pupils can work problems of the above types when arranged in a miscellaneous order.

5. For extra drill work, have pupils make up and solve problems of their own which involve the different types. Such problems are a good test of pupil's knowledge of the work covered.

19. **Types in Subtraction of Fractions That Involve Carrying:**

a. Fraction in Subtrahend Only:

$$\begin{array}{r} 10\,18 \\ -\ 27\frac{5}{6} \\ \hline \end{array} \qquad \begin{array}{r} 10\,18 \\ -\ \frac{5}{6} \\ \hline \end{array}$$

1. Teach the above type by borrowing method.

2. Do not permit pupil to put changes on paper.

b. Smaller Fraction in Minuend:

$$\begin{array}{r} 32\frac{1}{3} \\ -\ 19\frac{2}{3} \\ \hline \end{array} \qquad \begin{array}{r} 32\frac{1}{3} \\ -\ \frac{2}{3} \\ \hline \end{array}$$

1. Teach the above type by the borrowing method.

2. Train pupils to make all changes mentally.

3. As this is the most difficult type in the subtraction of fractions, pupils should receive a great deal of drill upon it.

4. Teach pupils to prove all answers to subtraction problems.

20. **Language Forms:**

a. When pupils can work readily any of the above types in the subtraction of fractions, the following language forms should be taught and applied:

a. **Find the difference between** $18\frac{2}{3}$ and $44\frac{1}{3}$. The difficulty lies in the fact that the smaller number is dictated first. Pupils are inclined to place as the minuend the first number stated in the problem.

b. **Find the difference between** $32\frac{2}{3}$ and $14\frac{1}{3}$.

c. **Subtract** $9\frac{3}{4}$ **from** $46\frac{1}{4}$.

d. **What must be added to** $24\frac{1}{4}$ to give $32\frac{3}{4}$?

21. **Subtraction of Fractions of Unlike Denominators:**

For drill material see text page 110.

$$\begin{array}{r} 24\frac{1}{4} = 24\frac{3}{12} \\ -\ 6\frac{2}{3} = 6\frac{8}{12} \\ \hline 17\frac{7}{12} \end{array} \qquad \begin{array}{r} \frac{7}{8} = \frac{35}{40} \\ -\ \frac{3}{5} = \frac{24}{40} \\ \hline \frac{11}{40} \end{array}$$

a. Notice the form of the work.

b. Use the language forms given under the subtraction of fractions of like denominators. See section 20.

c. Teach the reason this form of work is incorrect: $2\frac{3}{4} = \frac{9}{12}$.

MULTIPLICATION OF FRACTIONS.

The multiplication of fractions is to be taught by the cancellation method. The following is the order in which the different types are to be taught: For drill material see advanced text page 114, section 152.

a. $\frac{2}{3} \times \frac{5}{7} = ?$ (No cancellation.)

b. $\frac{3}{4} \times \frac{8}{9} = ?$ (Cancellation done diagonally.)

c. $\frac{3}{5} \times \frac{7}{14} = ?$ (Cancellation done vertically. Emphasize the fact that cancelling is simply dividing a number above the line and a number below the line by the **same** number. Show that for this reason we never cancel horizontally.) Text pages 119, 120, 121.

d. $24 \times \frac{5}{6} = ?$ (Teach that 24 is to be considered as the fraction $\frac{24}{1}$. Pupils often fail upon this type through regarding 24 as the denominator of a fraction.) Text pages 115–116.

e. $\frac{5}{8} \times 12 = ?$ (The same difficulty appears in this type as in Type *d*.)

22. Changing Mixed Numbers to Improper Fractions:

It is at this point that pupils should be taught to change mixed numbers to improper fractions. (See text page 95.)

Change $2\frac{3}{4}$ to an improper fraction.

a. First, teach pupils to change $2\frac{3}{4}$ to an improper fraction simply as a mechanical operation: $4 \times 2 = 8 + 3 = \frac{11}{4}$.

b. Later show pupils why this is done. Ask. "How many fourths are there in one whole? ($\frac{4}{4}$) Two wholes? Three wholes? etc."

c. In $2\frac{3}{4}$ we have the whole number 2 or $\frac{8}{4}$ and $\frac{3}{4}$ more or $\frac{11}{4}$.

d. Pupils should be able to readily change any mixed number to an improper fraction before the following types in the multiplication of fractions are introduced.

f. $2\frac{2}{3} \times \frac{6}{7} = ?$ (Pupils often attempt to cancel before changing the mixed number to an improper fraction. Use small numbers.)

g. $2\frac{3}{4} \times 24 = ?$ (There are two difficulties to be noted in this type. 1. Changing the mixed number to an improper fraction. 2. Regarding 24 as a correct fraction.)

h. $\frac{5}{8} \times 2\frac{2}{5} = ?$ (The same difficulty appears in this type as in Type *g*.)

i. $5\frac{5}{6} \times 4\frac{2}{3} = ?$ (Be careful while teaching the principle of this type to use small numbers.)

Suggestions:

1. Drill upon each of the above types until it has been mastered. Do not be in too great a hurry to introduce a new type. As each new type is mastered, review in a miscellaneous order all previous types.

2. There is no necessity for using large numbers when teaching the multiplication of fractions. **No number larger than 25 should be used.**

3. Insist upon all answers being reduced to lowest terms.

23. **Language Forms for Multiplication of Fractions:**

a. **Find the product of** $\frac{3}{4}$ and $\frac{7}{8}$.

b. **Multiply** $12\frac{5}{6}$ **by** $\frac{8}{9}$.

c. Find $\frac{6}{7}$ of 21. (Pupils should be able to illustrate this type with the aid of a diagram.) Text page 138, Sections 179 and 181.

d. Find $\frac{2}{3}$ of $\frac{5}{6}$.

e. What is $\frac{5}{8}$ of $6\frac{2}{3}$?

DIVISION OF FRACTIONS.

1. Have pupils read problems in the division of fractions and tell which fraction is the **divisor** and which the **dividend.**

2. Emphasize the fact that it is the **divisor** which is **inverted.**

3. Emphasize the fact that fractions may be cancelled only when they appear with multiplication sign between them.

4. Pupils should be taught **to prove their answers** to division problems.

5. The following is a list of the types of problems in the division of fractions. The types are listed in the order in which they are to be taught.

a. $\frac{3}{4} \div \frac{7}{8} = ?$

b. $\frac{7}{9} \div 9 = ?$

c. $24 \div \frac{2}{3} = ?$

d. $2\frac{2}{3} \div 6 = \frac{8}{3} \times \frac{1}{6} = \frac{4}{9}$

e. $6 \div 4\frac{2}{3} = ?$

f. $\frac{7}{8} \div 2\frac{2}{3} = ?$

g. $2\frac{3}{4} \div \frac{2}{3} = ?$

h. $4\frac{2}{3} \div 6\frac{3}{4} = ?$

After each of the above types has been taught, the work of sections 158, 168, 180 of text may be used as review work.

24. **Language Forms in Division of Fractions:**

The following are the language forms to be used in the work of the division of fractions:

a. **Divide** $\frac{3}{4}$ **by** $\frac{7}{8}$.

b. **How many times is** $1\frac{2}{3}$ **contained in 20?**

Pupils often fail upon this type because they do not realize which number is the dividend and which is the divisor. First teach the principle of the work by using integers.

25. **Given Part to Find Whole:**

Perhaps the most difficult of all problems in fractions is the following: 18 is $\frac{3}{4}$ of what number? (Text pages 59, 139, 140, 141, 142, 143, 144.) The solution should be as follows:

$\frac{3}{4}$ of the number $= 18$.

$\frac{1}{4}$ of the number $= 6$.

The number is 24.

Pupils should be able to show the meaning of the above statement by means of a diagram, also be able to tell why 18 is divided by 3, not 4, to find $\frac{1}{4}$ of the unknown number. Show pupils why the following form of work is incorrect: $\frac{3}{4} = 18$, etc.

b. Simplify: $\frac{137\frac{1}{2}}{200}$ Pupils should be first taught to read $\frac{137\frac{1}{2}}{200}$ as $137\frac{1}{2}$ divided by 200. As a second step have pupils write the problem thus: $137\frac{1}{2} \div 200 = ?$

c. What fraction of $\frac{3}{8}$ is $\frac{1}{8}$? (Text pages 131, 145, 146.)
$\frac{5}{8}$ is what fraction of $\frac{8}{9}$?
22 is what fraction of 22?

d. What is the ratio of 7 to 15? Of $\frac{2}{3}$ to $\frac{3}{4}$? Of 16 to 4? (Text pages 54, 55, 56, 91–92. Pages 130, 19–20. Pages 145–146.)

QUESTIONS ON FRACTIONS.

26. The following questions and the tests of the next section will show in a general way the knowledge of fractions which a child should have upon completing the work of this topic. These questions will also serve as a guide for Normal students desiring to prepare themselves for a test in the subject of fractions. Questions marked with a star are for teachers only.

1. Illustrate two different types of improper fractions.
2. Show with aid of a diagram that $\frac{4}{8}$ or $\frac{3}{6}$ of a line is the same as $\frac{1}{2}$ of the line.
3. May $\frac{7}{9}$ or $\frac{2}{7}$ be changed to 12ths? Give a reason for your answer.
4. Illustrate with diagrams what is meant by each of the following statements:
   a. Find $\frac{2}{3}$ of \$12. *b.* \$12 is $\frac{2}{3}$ of what amount? *c.* $\frac{1}{5}$ is what fraction of $\frac{4}{5}$? *d.* Find $\frac{2}{3}$ of $\frac{3}{4}$ of a line.
5. Which of the following fractions may be reduced to lower terms: $\frac{7}{9}$, $\frac{10}{15}$, $\frac{1}{2}$, $\frac{11}{12}$? How can you tell?
6. When 2 is divided by $\frac{3}{4}$, why is the answer greater than 2?
*7. In changing $\frac{27}{5}$ to a mixed number, why is 27 divided by 5?
*8. In changing $3\frac{2}{3}$ to an improper fraction, why is the whole number, 3, first multiplied by the denominator of the fraction, $\frac{2}{3}$?
*9. Illustrate the difference between a prime number and an odd number.
10. Why must unlike fractions be changed to fractions of the same denominator before they may be added?
11. Read $\frac{19}{4}$ in two ways.
12. Which of the following are in a correct form to be cancelled? Change those that are not, so that they may be cancelled.

a. $24 \times \frac{2}{3} = ?$	*d.* $\frac{3}{4} \times \frac{7}{8} = ?$
b. $2\frac{2}{3} \times \frac{5}{6} = ?$	*e.* $\frac{3}{7} \times 2\frac{1}{2} = ?$
c. $8\frac{2}{3} \times 9\frac{1}{2} = ?$	*f.* $\frac{3}{7} \times 14 = ?$

13. How many thirds, fourths, sixths, are needed to make a whole?
14. \$18 is $\frac{3}{4}$ of what amount? Show with the aid of a diagram why \$18 is divided by 3 to find $\frac{1}{4}$ of the unknown amount.
15. State and solve a problem of your own in which it is necessary to invert one of the fractions.

16. Tell what is wrong with each of the following problems and rewrite each problem in its correct form:

a.	*b.*	*c.*
$3\frac{1}{2} = \frac{1}{24}$	$6\frac{3}{4}$	$6\frac{4}{8}$
$2\frac{1}{3} = \frac{8}{24}$	$9\frac{2}{4}$	$\frac{3}{8}$
$6\frac{3}{8} = \frac{9}{24}$	$6\frac{1}{4}$	$2\frac{3}{8}$
$12 \quad \frac{29}{24} = 1\frac{5}{24}$	$21\frac{6}{4}$	$9\frac{2}{8}$

17. In the multiplication of fractions, tell in what directions numbers may be cancelled. Is it possible to cancel horizontally? For instance, can you cancel the 4 and 8 in $\frac{4}{5} \times \frac{8}{11} = ?$ Give a reason for your answer.
18. Write two mixed numbers and change them to improper fractions.
19. Find the smallest common denominator for the following fractions and show all of your work:
    a. $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{5}$, $\frac{2}{7}$.
    b. $\frac{5}{8}$, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{6}{12}$, $\frac{3}{9}$.
20. Correct the following statement:
    $4\frac{2}{3} = \frac{4}{6}$
21. In the fraction $\frac{7}{12}$, which term is the numerator and which term the denominator of the fraction? What does each term tell?
22. Change the following fractions to 36ths: $\frac{4}{12}$, $\frac{1}{9}$, $\frac{3}{4}$.
23. Does the value of a fraction increase as its denominator increases? Is $\frac{4}{17}$ of a line greater or less than $\frac{4}{19}$ of a line?

SUMMARY OF THE FORMAL WORK IN FRACTIONS.

27. The following tests in the formal work of fractions are intended for pupils of the grammar grades who have completed the work in fractions. Inexperienced teachers will find these tests useful as guides when planning reviews. The tests will also serve to indicate to Normal students some of the fundamental facts in fractions with which they must be familiar.

Test in Addition and Subtraction of Fractions.

1. Reduce to lowest terms: $\frac{19}{57}$, $\frac{231}{432}$, $\frac{430}{855}$, $\frac{92}{112}$, $\frac{13}{26}$.
   What are the tests as to whether or not fractions may be reduced by dividing by 2, 3, 5, or 10?
2. Classify each of the following as proper or improper fractions or mixed numbers: $\frac{7}{8}$, $1\frac{3}{4}$, $\frac{9}{9}$, $\frac{11}{3}$, $\frac{6}{7}$.
3. Write two proper fractions which may be reduced to lower terms. Reduce your fractions.
4. *a.* Write two improper fractions. Change them to mixed numbers.
   b. Write two mixed numbers and change them to improper fractions.
5. Find the sum of: $3\frac{3}{10}$, $8\frac{5}{9}$, $2\frac{1}{6}$, $3\frac{1}{5}$, $\frac{3}{4}$, $2\frac{5}{18}$, $2\frac{1}{2}$, $8\frac{1}{3}$. Show all of your work for finding the common denominator.
6. Show by a diagram that $\frac{3}{6}$, $\frac{4}{8}$ and $\frac{1}{2}$ of a line are equal.
7. Change $\frac{3}{8}$, $\frac{5}{12}$ to 24ths.

8. Find the difference between $20\frac{2}{3}$ and $408\frac{1}{4}$.
9. Subtract $12\frac{3}{7}$ from 25.
10. What must be added to $206\frac{5}{6}$ to make $1000\frac{1}{6}$?
11. $206\frac{2}{3} - 18\frac{2}{3}$.

Test in Multiplication and Division of Fractions.

1. Find $\frac{3}{4}$ of $20\frac{2}{3}$.
2. What is $\frac{1}{2}$ of $\frac{7}{9}$?
3. How many times is $\frac{2}{3}$ contained in $2\frac{3}{4}$?
4. Divide $\frac{7}{8}$ by $1\frac{2}{3}$.
5. $\$2\frac{3}{4}$ is $\frac{7}{8}$ of what amount?
6. Divide 1 ft. 9 in. by 7 inches.
7. $\frac{137\frac{1}{2}}{300} = ?$
8. 17 is what fraction of 29?
9. What part of $\frac{7}{9}$ is $\frac{2}{9}$?
10. $\frac{3}{5}$ are what part of $\frac{7}{9}$?
11. What is the ratio of 7 to 11? Of 12 to 4? Of $\frac{3}{4}$ to $\frac{4}{5}$?
12. If $\frac{4}{5}$ of a yard of satin is worth \$1.60, what is the price of satin per yard?
13. What fraction of \$2.10 is \$1.05?
14. *a.* $12\frac{3}{4} \times 7\frac{2}{3} = ?$ *b.* $3\frac{1}{2} \div 24 = ?$ *c.* $\frac{7}{8} \div \frac{2}{3} = ?$

HOW TO TEACH THE CRITICAL DIFFICULTIES OF DECIMALS.

This summary of type problems in decimals and the suggestions made here in regard to the form and the method of work to be employed in the teaching of this subject may be used in connection with the indicated sections of the California State Series Advanced Arithmetic or any other arithmetic text.

Normal students will be expected to be familiar with the difficulties, form of work and methods of teaching suggested in the following sections.

28. Reading and Writing Decimals:

1. Teach pupils to read and write decimals through millionths' place.

2. Be careful to see that pupils pronounce and spell correctly the names of the decimal places. In writing the word hundredths, pupils often omit the second d. In pronouncing the word hundredths, pupils often neglect to sound the th. Point out that 100 and one hundredth are not the same. The fourth and fifth decimal places are "ten-thousandths; hundred-thousandths," not tens of thousandths or hundreds of thousandths.

3. Teach that the hyphen between the ten and thousandths in the word **three ten-thousandths,** makes the two words one. 3 ten-thousandths is written decimally as .0003. The name of the decimal place is ten-thousandths. The problem requires us to write three of them.

When there is no hyphen between the ten and thousandths, they are to be regarded as separate words. Thus, ten thousandths would be

written decimally .010. The name of the decimal place is thousandths. The problem requires us to write ten of them.

4. In reading a number such as 273.003, there is only one **and.** This is the decimal point. The above number should be read "two hundred seventy-three **and** three thousandths." Have pupils read $20.85, 1.04, etc.

5. Ask: What is the name of the third place after the decimal point? The fourth place? The second place, etc. How many decimal places are needed to express ten-thousandths? Hundredths? etc.

6. When writing decimals from dictation, insist upon pupils placing the decimal point as soon as the "and" is mentioned. This is the only way in which pupils will acquire the habit of always inserting the decimal point.

7. Teach which is of the greater value and why—3 tenths, 3 hundredths or 3 thousandths, etc. Pupils often think that three hundredths or three thousandths of a quantity is greater than three tenths of the same quantity.

8. In a number such as 2.2222, have a pupil tell the value of each 2 as compared with the 2 to its right or left.

9. Show pupils that .4, .40, .400 of a quantity are all of equal value. Point out the value of the naught in numbers such as the above.

10. Show pupils how the naughts change the value of each of the following numbers: .002, .02, 1.02, 1.10.

11. Explain that our system of money is a decimal system. (Text, pages 9, 14.)

12. Have pupils see that .6 is only another way of writing the fraction $\frac{6}{10}$. Emphasize the fact that a decimal is a **fraction** of a quantity.

ADDITION OF DECIMALS.

The addition of decimals is merely an application of the reading and writing of decimals.

Dictate exercises in which some of the numbers are hundredths, others thousandths, integers, etc. Do not permit pupils to fill out vacant places with naughts. Occasionally write the decimals out in words and have pupils add the numbers.

(For drill work, see Text page 18. Teach also the work of page 14.)

29. Language Forms:

The language forms to be used in the addition of decimals are:

a. Add the following: .003, .2, 1.46, 83, .004.

b. Find the sum of: 1.14, 89.2, 10006, 1.0012.

c. Work in the addition of decimals may be varied by using the following form:

Write in the order of their value and add: Sixteen; thirty-two and four hundred-thousandths; four hundred-thousandths; one and three millionths; eight hundredths. The above exercise is a good test *a.* of the pupil's ability to write decimals; *b.* to ascertain their relative value.

c. Emphasize the fact that in the addition of decimals, the decimal points must be kept under each other.

SUBTRACTION OF DECIMALS.

28. Types in Subtraction of Decimals:

a.	*b.*	*c.*	*d.*	*e.*	*f.*
275.85	275.85	275.85	.7	295	296
−14.25	−14.	− .974	− .0024	− 14.006	− .009

Little or no difficulty will be experienced in teaching the subtraction of decimals provided pupils are proficient in the reading and writing of decimals. Emphasize the fact that decimal points must be kept under each other.

29. Language Forms for Subtraction of Decimals.

Often pupils, who experience little difficulty in solving subtraction exercises in which the minuend and subtrahend are correctly placed, fail when confronted with a problem requiring discrimination between the two terms. As this is the form in which subtraction is usually encountered in life and even in the problems of a textbook, it is necessary to teach the various language forms which serve to indicate that the process of subtraction is to be employed. Only the more common forms are enumerated below in the order of their difficulty:

1. Take 4.06 from 19.65.
2. Subtract 82.3 from 963.64.
3. Find the difference between 444.85 and 200.
4. Find the difference between .245 and 48.65.

   (This last type is especially difficult because the smaller number is given first. Pupils often attempt to subtract the numbers in the order in which they appear in the problem.)
5. What must be added to 8.56 to make 27.2?
6. Teach Text page 25.
7. The addition and subtraction of decimals may be taught in a very short time provided pupils can read and write decimals without hesitation.

MULTIPLICATION OF DECIMALS.

30. Two Principles to Be Taught:

There are two things to be emphasized in teaching the multiplication of decimals — (1) the correct placing of the decimal point in the answer. (2) The correct placing of partial products with the different types of multipliers.

31. The Placing of the Decimal Point:

1. Do not insist upon pupils writing the multiplier so that its decimal point is under the decimal point of the multiplicand. This form often causes pupils to subtract instead of multiplying the exercise. Use this form:

2.4 2 9 6 × .9 6	not	2.4 2 9 6 ×.9 6

2. Do not use multiplicands or multipliers which have a great many figures. The fatigue induced by the working of such long problems leads to many mechanical errors. Two short problems are better than one long one for teaching a new principle.

TYPES OF MULTIPLIERS AND MULTIPLICANDS.

The type problems of multiplication are enumerated below in the order of their difficulty.

The same general form and method of work should be employed in teaching pupils to place their partial products when using each and every type of multiplier. The one rule, "Place your first answer directly under the number you have multiplied by" gives excellent results.

32. Types of Multipliers:

a.	*b.*	*c.*	*d.*	*e.*	*f.*
24.82	900.2	427.08	6.347	14.2	.007
$\times$.422	$\times$ 2.5	$\times$6.009	$\times$ 4.80	$\times .06\frac{2}{3}$	$\times$ .003

1. In type *a*, the emphasis is only upon the correct placing of the decimal point in the answer.

2. In type *b*, the naughts of the multiplicand offer a difficulty to many pupils.

3. In types *c* and *d*, pupils often fail to place the partial products in their correct place. Use the suggestion of "Place your first answer directly under the number you have multiplied by" and the difficulty will disappear.

4. In type *e*, be careful to see that pupils do not consider the fraction as occupying a decimal place and also that the product obtained by multiplying by 6 is correctly placed.

5. In type *f*, teach pupils that naughts must be added in the product to the *left* of the integer. Show why naughts must be added to the left and not to the right of the integer. Pupils often subtract instead of multiplying this type of problem as its appearance is very much that of a subtraction exercise.

33. Language Forms To Be Used In Connection With the Multiplication of Decimals:

1. Find the product of 24.84 and .32.
2. What is six hundredths of 24.84?
3. Find decimally $\frac{6}{10}$ of $\frac{245}{1000}$.
4. Multiply .002 by .003.

DIVISION OF DECIMALS.

In teaching the division of decimals, follow closely each of the following steps:

1. Be certain that pupils know the difference between the terms **dividend** and **divisor.**

2. Write on the board division problems in which there are more decimal places in the dividend than in the divisor. Write other problems in which there are **less** decimal places in the dividend than in the divisor.

3. Teach that there must always be at least as many decimal places in the dividend as in the divisor. Emphasize the fact that the dividend may have more decimal places than the divisor.

From the problems on the board, have pupils select those in which the relation between the number of decimal places in the dividend and the divisor is correct.

4. Teach pupils that naughts are added to dividends that need more decimal places. Show why these naughts do not change the value of the decimal.

5. Dictate many problems to pupils and have them fix the dividends that need attention. Do not leave this step of the work until it is thoroughly mastered.

6. Placing Decimal Point in Answer:

Use the following method to teach pupils to place decimal points in the answer. Insist upon pupils placing the decimal point in the answer before the problem is solved.

a. Write the following problem on the board:

.0 0 2) 4 1 8.2 8 4 9

b. Place your chalk on the decimal point in divisor and count the number of decimal places, saying "decimal point—one, two, three." Place your chalk on the decimal point in the dividend and count off the same number of decimal places, saying "decimal point—one, two, three." Place decimal point in quotient above and after the last number you counted. Try other types and use the same language form.

.4) 8 2.4 2 9

c. Dictate problem after problem and have pupils count off the decimal places and fix the decimal point in the quotient. At this stage of the work, do not permit pupils to solve the problems. All emphasis should be upon the correct placing of the decimal points.

d. The following are the advantages of the above method: (1) It is not necessary for pupils to memorize any rule for pointing off the answer in division of decimals. Memorized rules are soon forgotten. (2) The decimal is placed before beginning any of the work of division; (3) when the decimal point is placed in the answer, pupils can tell at a glance the number of places to which the answer has been carried.

34. Carrying Answers Out to Three Decimal Places:

1. When pupils can correctly place the decimal point in any type problem, show them that the problem is really one in the division of integers.

2. Teach pupils that to carry an answer out three places decimally, means to have three figures in the answer after the decimal point. Pupils sometimes think that adding three naughts to the dividend has carried the answer out decimally three places.

3. After carrying the answer out to three decimal places, a plus sign should be used, not a common fraction remainder.

Suggestions:

1. Insist upon pupils always placing the decimal point in the answer before attempting to perform the division of the problem. This is the only way in which pupils will ever acquire the habit of always placing the decimal point in the quotient.

2. Have all answers carried out to three decimal places.

3. Do not place the emphasis upon the length of the problem, but upon the correct placing of the decimal point.

4. Occasionally make use of the divisor 3.1416 which will be used later in the work of mensuration.

5. Use the following language forms when dictating problems:

35. Language Forms for Division of Decimals.

a. How many times are .2 contained in 42.006?

b. Divide 4.09 by .6.

c. Divide 2 by 11.

d. $20.45 is .006 of what amount?

36. Types to be Found in the Division of Decimals.

a.	*b.*
.03) 24.829	6.5) .0042
c.	*d.*
.83) 4.92	.07) 63.2
e.	*f.*
.004) 923	27) 1.0246
g.	*h.*
Divide 2 by 7.	Change $\frac{19}{4}$ to a decimal.
i.	*j.*
$1.05) $728.45	3.1416) 22

For drill work after all the above principles have been taught see Text, pages 68–69–70.

FRACTIONAL EQUIVALENTS.

Have the pupils memorize the following:

$.50 = \frac{1}{2}$	$.40 = \frac{2}{5}$	$.12\frac{1}{2} = \frac{1}{8}$
$.33\frac{1}{3} = \frac{1}{3}$	$.60 = \frac{3}{5}$	$.37\frac{1}{2} = \frac{3}{8}$
$.66\frac{2}{3} = \frac{2}{3}$	$.80 = \frac{4}{5}$	$.62\frac{1}{2} = \frac{5}{8}$
$.25 = \frac{1}{4}$	$.16\frac{2}{3} = \frac{1}{6}$	$.87\frac{1}{2} = \frac{7}{8}$
$.75 = \frac{3}{4}$	$.83\frac{1}{3} = \frac{5}{6}$	$.10 = \frac{1}{10}$
$.20 = \frac{1}{5}$	$.14\frac{2}{7} = \frac{1}{7}$	$.30 = \frac{3}{10}$

In connection with this section teach the material on pages 156, 157 and 158 of text.

37. To Divide Integers and Decimals by 10, 100, 1000.

The following are the types of numbers which pupils should be taught to divide first by 10, then 100, then 1000 (text pages 37, 40): 135, 40, 400, 6, .2, .06, 1.3, $4, $200, $1.60, $.85, $.40.

a. Write 4.6 on the board. Erase the 6. This leaves 4 with a decimal point after it. Have pupils read this number.

b. Write 16, 18, 32 on the board. Have pupils locate the decimal point in each of these numbers.

c. Teach that to divide 16, 18, 32, etc., by 10, the decimal point is moved one place to the left.

d. Teach that numbers, such as 50, 60, 40, etc., are divided by 10 by moving the decimal point one place to the left. Emphasize the fact that such numbers are divided by 10, not by crossing off the naughts but by moving the decimal point one place to the left.

e. Care must be taken to see that pupils do not make mistakes in dividing numbers such as $.40, $1.40, etc., by 10. Ask: "Does crossing off the naught divide the above numbers by 10?" "How are they divided by 10?"

f. Ask: "How is a number divided by 10?"

g. When the decimal point has been moved one place to the left, what has been done to the number?

h. Repeat the same steps in teaching pupils to divide numbers by 100 and 1000. Use the types listed above.

i. Ask: "How are numbers divided by 10, 100, 1000?" When decimal points are moved one, two or three places to the left, what has been done to the number?

38. To Multiply Numbers by 10, 100, 1000.

a. Teach that numbers are multiplied by 10, 100, 1000 by moving the decimal point one, two, or three places to the right.

b. Ask: "How is a number multiplied by 10? 100? 1000?"

c. When is the decimal point moved to the right?

d. When is the decimal point moved to the left?

e. Put a column of numbers on the board. Mix in decimals with the integers. Call upon pupils to give the answers when the various numbers are divided by 10, 100, 1000, or when multiplied by 10, 100, 1000.

MISCELLANEOUS QUESTIONS ON DECIMALS.

After all of the principles in decimals have been taught, the following questions and the tests of the next section may be used to ascertain the extent of the pupils' knowledge.

Normal students will find these questions and tests useful in their preparatory work in arithmetic.

1. Write in their order the names of the decimal places through millionths.

2. Explain why our system of money is a decimal system.

3. Which is of the greater value, $1.10 or 1100 mills? How much greater?

4. Write in figures three hundred thousandths; three hundred-thousandths; three hundred thousand. How do you distinguish each type?

5. In the expression 111.111, how does each 1 compare in value with the one to its left?

6. If a naught is added to the right of \$2.10, has the number been multiplied by ten? If the naught at the end of \$2.10 is crossed out, has the number been divided by ten? Explain in full.

7. What is the difference in value between 100 and .01?

8. Where is the decimal point in a number such as 24?

9. If the decimal .9 is a fraction, where is its numerator and denominator?

10. In a decimal expression such as $.66\frac{2}{3}$, does the fraction, $\frac{2}{3}$, count as a decimal place?

11. Show that .8, .80, .800 of a quantity are all of equal value.

12. Write the table of United States money.

13. In the multiplication of decimals, how do you know the number of decimal places to point off in the answer?

TEST IN ADDITION AND SUBTRACTION OF DECIMALS.

1. Express each of the following decimals as common fractions: .024, .004, .8, 2.101, .400, .00265.

2. If two naughts are added to the right of the 1 in the decimal expression .1, what is the effect upon the value of the decimal? If two naughts are added to the left of the 1 in what way is the value of the decimal .1 changed?

3. How does .4 compare in value with .44 and with .004?

4. Show that .1, .10, .100 are of equal value.

5. What is the name of the fourth decimal place? The third decimal place? The second decimal place?

6. Write at least five aliquot parts of a dollar.

7. Arrange the following decimals in the order of their value and add: Six hundred six and six hundred-thousandths; eighteen and one millionth; twelve and six tenths; five hundred seven thousandths; six hundred; fifteen hundredths.

8. Find the difference between .1 and 1.426.

9.

a.	*b.*
296.4	\$110
— 18.008	— \$ .85

10. Find out by changing to common fractions whether you would rather have .5 of a pie, .50 of a pie, or .500 of the same pie.

11. What is the difference in value between (*a*) 1 and .1; (*b*) 1. and .01; (*c*) 1. and .001; (*d*) 100 and .01 ?

12. Place the decimal point in each of the following numbers: sixteen hundredths; eighty-five; five thousandths; two hundred seventy-five; sixteen and four thousandths; twelve.

13. Write decimally and add: four dollars and sixty-five cents; twenty-five and seven tenths dollars; forty-eight dollars and nine

cents; eight thousand three hundred sixteen dollars and sixteen cents four mills; four and sixteen hundredths dollars; eight dollars and four cents; nine mills; three and two hundred fifteen thousandths dollars.

14. What is .1 of a dollar? One hundredth of a dollar? One thousandth of a dollar?

TEST IN MULTIPLICATION AND DIVISION OF DECIMALS.

1. Find the product of 80.04 and 40.8.

2. Divide each of the following problems in the shortest way possible:

$\$1.10 \overline{)\ \$240}$ $\quad 300 \overline{)\ 150.1}$ $\quad 400 \overline{)\ .8394}$

$\$600 \overline{)\ \$84.80}$ $\quad \$.10 \overline{)\ \$1.10}$ $\quad .40 \overline{)\ .400}$

3. Change $\frac{19}{21}$ to a decimal. (Carry quotient to three decimal places.)

4. Multiply .004 by .003.

5. Place only the decimal point in the quotient of each of the following problems:

a. $3.1416 \overline{)\ .259}$ $\quad$ *b*. $.003 \overline{)\ 2.429}$ $\quad$ *c*. $.019 \overline{)\ .4}$ $\quad$ *d*. $34 \overline{)\ 802.34}$

e. $.7 \overline{)\ 84}$ $\quad$ *f*. $19 \overline{)\ .1}$ $\quad$ *g*. $\$.01 \overline{)\ \$1}$ $\quad$ *h*. $\$1 \overline{)\ \$1.01}$

6. Divide 2 by 9.

7. Multiply each of the following numbers in the shortest way possible, first by 10, then 100, and then 1000:

$\$.45$, $\$1.40$, .0008, .6, .94, $\$1.008$, 2.75, .009, 46., $\$.05$.

8. Find decimally $\frac{7}{10}$ of 42.094.

9. What is $6\frac{2}{3}$ hundredths of 245.6?

10. Write each of the following as decimals: $\frac{3}{8}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{2}{3}$, $\frac{1}{7}$, $\frac{1}{9}$, $\frac{5}{8}$, $\frac{1}{8}$, $\frac{3}{4}$, $\frac{1}{6}$.

RATIO, AVERAGE, TABLES, BILLS AND RECEIPTS.

The following questions cover several miscellaneous topics. The questions will indicate the principles to be taught under each topic.

39. Test on Ratio and Average.

1. What is the ratio of a pound to an ounce?

2. From the following table, find Fred's average gain in weight and in height each year:

Age.	Height.	Weight.
11 yr.	4′ 11″	98 lb.
12 "	5′ 2″	104 lb. 4 oz.
13 "	5′ $3\frac{1}{4}$″	107 lb. 9 oz.
14 "	5′ 4″	110 lb. 1 oz.

a. Be certain that pupils first find the gain in height.

b. Since there were 3 gains, the actual gain is divided by 3 to find the average gain in height.

3. The expenses of a picnic were $3.40. Divide this amount between Mrs. Brown and Mrs. White in the ratio of 2 to 3.

4. Miss Saunders had 57 pupils in her class. There were 42 pupils in Miss Ryan's class. Find the ratio of the number of pupils in Miss Saunders' class to the number in Miss Ryan's class.

5. In 1890 the population of San Francisco was 298,997. In 1906 the population of San Francisco was 450,000. Find the ratio of the population in 1906 to the population in 1890.

6. During the month of September Mrs. Fisher used 645 cubic feet of gas; in October, 750 cubic feet; in November, 1,200 cubic feet; in December, 1,950 cubic feet; in January, 2,645 cubic feet. Find the average gain in the number of cubic feet of gas used each month.

7. What is the ratio of $\frac{2}{3}$ to $\frac{3}{4}$?

8. Is the ratio between two quantities ever expressed as a concrete number? Why?

9. Is the ratio of 7 to 9 the same as the ratio of 9 to 7?

10. During a certain year the rainfall for six months in San Francisco was as follows: December, 5.11 inches; January, 4.98 inches; February, 3.72 inches; March, 3.28 inches; April, 2.14 inches; May, 0.68 inches. Find the average rainfall for each month.

40. Tables: Linear, Square, Weights, Country, Time, Liquid.

a. How many pints of milk may be poured into a five-gallon can?

b. Mr. Bray owns a section of land. What is the value of the entire farm if one acre is worth $75?

c. What is the value of 956 eggs at $.32 a dozen?

d. Helen practices two and one half hours a day on the piano. How many minutes does she practice each day?

e. At $11.50 per ton, what is the value of 16,480 pounds of hay?

f. I bought $2\frac{1}{2}$ pounds of sugar. How many ounces of sugar did I purchase?

g. What is the cost of a great gross of pencils if one pencil costs $1\frac{1}{2}$¢.

h. Mr. Brown owns property having a frontage of $\frac{1}{5}$ of a mile. Express the width of the lot in feet.

i. A square foot is how many times as great as a square inch?

j. Which is the greater in value, $2.10 or 8,124 mills? How much greater?

k. How many steps, each 12 inches long will a man take in walking one mile?

l. What is the length and width in feet of a square room, the floor of which has an area of one square rod?

m. Find the sum of $4'\ 6''$; $\frac{5}{8}''$; $1'\ \frac{1}{4}''$; $2'\ 7\frac{1}{2}''$.

n. Find the difference between $102\frac{1}{4}'$ and $\frac{5}{8}''$.

o. How many dozen are there in a gross? Ounces in a pound? Pounds in a hundredweight? Minutes in an hour? Units in a score? Quarts in a gallon? Pints in a quart? Mills in a cent? Pounds in a short ton? Hundredweight in a long ton?

II.

a. If a dealer has 288 pencils in stock, are there enough to fill an order for a gross of pencils?

b. Which is the larger amount, 18 cents or 25 mills?

c. How many pounds are there in a long ton? How many cwt. are there in 3485 pounds?

d. Alice practiced 85 minutes on the piano. How much more or less than an hour did Alice practice?

e. How many pint bottles may be filled from a gallon of milk?

f. Last Christmas I sent to New York a package weighing 22 ounces. How much more or less than a pound did the package weigh?

g. How many cubic inches are there in a gallon?

h. How many cords of wood are there in 15,744 cubic feet of lumber?

i. How many pounds of coal are there in 2 hundredweight?

j. How many minutes are there in 2 hours 45 minutes?

k. How many years are there in "four score and seven years"?

l. How many hundredweight are there in one ton?

m. If a ton of coal is worth $14, what is the value of three hundredweight of coal?

n. A gallon of water is the same as how many cubic inches of water?

41. Bills and Receipts. (Text pages 42–44.)

1. Mr. Berryman rented a house at 1207 Sacramento Street from Mr. J. R. Hansen for the sum of $35 a month. On May 1st, Mr. Berryman paid the rent for one month in advance. Write the receipt Mr. Hansen gave to Mr. Berryman.

2. Make out and receipt the following bill: Mrs. J. H. Moore, residing at 3746 Sacramento Street, bought of her dealer the following: January 3, 1914, 4 yd. blue ribbon @ $.30; 2½ yd. lace @ $.75; 6 handkerchiefs @ $.25; ¾ yd. lace yoking @ $2.25.

3. Make out a receipt of your own showing that money has been paid by you to some other person for work done.

4. Make out and receipt the following bill of goods purchased from your grocer by you on May 1st:

3 lb. coffee	@	$.35
1 lb. tea	@	.80
2 doz. eggs	@	.30
1 roll of butter	@	.65

5. Mrs. Adams bought from Mrs. Dean a sewing machine for the sum of $60. Write a receipt showing that Mrs. Dean has received the money from Mrs. Adams.

AREA OF RECTANGLES.

(Text pages 80–84.)

The material of pages 80, 81, 82, 83 of the Advanced Arithmetic may be supplemented with much additional work from Bulletin No. 31, pages 1–26.

42. Test on Area of Rectangles. (Text pages 80–84.)

The following questions may be used to test a pupil upon the work of pages 80 to 84 of text.

1. Draw three different figures, each of which is a rectangle. Label the base and altitude of each figure and shade its surface.
2. Illustrate with diagrams the difference between a horizontal rectangle and a square.
3. Illustrate with diagrams the difference between two square inches, two inches and a two-inch square.
4. Draw on your paper four right angles, each of which is in a different position.

5. Is a very thin sheet of tissue paper a surface or a solid? Give a reason for your answer.
6. Illustrate by means of a diagram which is the larger, a square mile or a section of land.
7. A lot is 125 feet deep and has a frontage of 25 feet. What is the perimeter of the lot?
8. How many feet of picture molding are needed for a room 21 feet long and 16 feet wide?
9. Make a list of three rectangular surfaces that appear in your classroom.
10. Find the perimeter and the surface of this page of your book.
11. A forest extends over two sections of land. How many acres of land are covered by the forest?
12. What is the length in feet of the side of a square field, the area of which is one acre?
13. Draw two oblique, two horizontal and two vertical parallel lines.
14. Draw a square foot on your paper. (Let a line 6 inches long represent a foot.) Divide your square foot into the greatest possible number of square inches. What is the answer?
15. Draw a square yard on your paper. (Let 6 inches represent 1 yard.) Show by your diagram the number of square feet there are in one square yard.
16. What is the perimeter and what is the area of a field one mile square? Diagram your field.
17. Tell for what, in the square measure or the linear measure, each of the following numbers stands: 640, 3, 12, 9, 320, 160, $30\frac{1}{4}$, $208\frac{3}{4}$, $16\frac{1}{2}$, 144, $5\frac{1}{2}$, 5,280.
18. With a diagram, show the number of square inches there are on the surface of a three-inch square. What is its area?
19. Draw, if you can, a rectangle which contains no right angles.
20. Illustrate with marked diagrams the difference between the perimeter and the surface of a four-inch square.
21. Express $14\frac{2}{3}$ square feet as square inches. Change 4 acres to square rods.
22. Draw a horizontal rectangle, a vertical rectangle and a square. Label the base and altitude, and shade the surface of each figure.
23. Draw a line which is perpendicular to another line.
24. If the area of a rectangle is given in acres and the length of one side in rods, how is the perimeter of the rectangle found? Illustrate with a problem of your own.
25. If the area of the floor of a square room is one square rod, what is the length and width of the room expressed in feet?
26. At $175 an acre, what is the value of a field which is 400 rods long and $\frac{1}{4}$ of a mile wide?
27. How many rails, each $16\frac{1}{2}$ feet long, are needed to enclose a square field, the area of which is one square mile?

28. A field contains 3 acres of land and is $\frac{1}{2}$ of a mile long. What is the width of the field?
29. If the area of a field is 10 acres and one side is 20 rods long, what is the perimeter of the field?
30. May an acre or any part of an acre ever be used to measure the length of a line? Give a reason for your answer.
31. Mr. Jones paid \$550 for his farm. Each acre cost him \$55. If the length of the farm is 180 rods, what is its perimeter?

CUBIC MEASURE.

(Text pages 84–86.)

The material of pages 84, 85, 86 of the text may be supplemented with much additional work from pages 55 through 68 of Bulletin No. 32.

43. Test on Cubic Measure. (Text pages 84–86.)

The following questions will serve as a test of a pupil's knowledge after he has completed pages 84 through 86 of text.

1. How many cords of wood are there in a pile which is 12 feet long, 8 feet wide and 4 feet high?
2. How many cubic yards of dirt were removed from a lot $114' \times 50'$ if the earth was removed to a depth of 6 feet?
3. A rectangular tin can is $8'' \times 10''$ and 16 inches high. How many gallons of water will the can hold?
4. A swimming tank is 110 feet long, 75 feet wide and has a depth of 5 feet. How many gallons of water are there in the tank when it is one half full? ($7\frac{1}{2}$ gal. = 1 cu. ft.)
5. Illustrate with a drawing the difference between a two-inch square and a two-inch cube.
6. How many faces has a cube? What is the volume of a three-inch cube?
7. Is a sheet of paper a surface or a solid? Why?
8. In order to find the volume of a prism, how many dimensions must be known? State and solve a problem of your own in which you are required to find the volume of a prism.
9. A gallon of water is the same as how many cubic inches of water?
10. If the length and width of a room are given, tell whether or not you can find (*a*) the area of the ceiling, (*b*) the volume of air in the room, or (*c*) the number of feet of picture molding needed for the room.
11. Mr. Harris removed one foot of dirt from a lot having a depth of 114 feet and a frontage of 75 feet. How many cubic yards of earth were removed from the lot?
12. What is the volume of a four-inch cube?
13. A rectangular vessel is 26 inches long, 18 inches wide and 9 inches deep. What is its capacity expressed in gallons?
14. Write the table of cubic measure.

PERCENTAGE.

(Text pages 166–190.)

The work of each of the three cases in percentage may be divided into two parts—formal work and problem work. Bulletin No. 29 presents, in the order of their difficulty, each of the points to be taught under the formal and problem work of each of the cases in percentage. As this bulletin is for self-instruction, it contains all necessary explanations of difficulties. Normal students are expected to be familiar with all of the problems of this bulletin as well as the problems in the State Text on pages 166–190.

The mechanical or formal work of each case in percentage should be mastered before the problem work is introduced. For this reason, the formal work of each case of percentage is taught before pupils are required to center their attention upon the problem work of a given case. Most of the complicated and impractical problems which fall under the second case in percentage have been omitted from Bulletin 29.

43. Formal Work of Case One in Percentage.

a. The most difficult piece of formal work in the first case in percentage is the changing of the various types of per cents to correct decimals.

b. Pupils should be taught to change each type of per cent as a separate step of work. As soon as possible, work should be given which will be an application of the type taught. Unless this is done, pupils will think of the changing of per cents to decimals as a line of work separate from the rest of percentage.

c. Bulletin No. 29, pages 3 through 23, presents in the order of their difficulty, each of the difficulties that occur in the formal work of Case One.

d. The following are the most difficult sections: 7, 8, 11, 12, 16, 18, 22, 25, 26, 28, 29, 30. Teachers should question pupils closely upon each of these sections before permitting them to take up any new work.

e. The tests of sections 32 and 33 may be used as guides when making out tests upon the formal work of Case One.

Problem Work of Case One in Percentage.

The failure of many pupils in the problem work of arithmetic may be traced (1) to their inability to read a problem and ascertain what is given in the problem and what is required to be found; (2) to lack of ability to understand the situation, words and phrases of the problem; (3) helplessness in attempting the solution of the problem; (4) failure to test the answers to problems.

44. Statement of Problems:

Pupils should be required to write a statement of every problem before attempting its solution.

When a pupil can write a correct statement of what is given in a problem and what is to be found, it is an indication to the teacher that the pupil has at least read the problem. The method of teaching

pupils to write the statement of a problem is to be found on page 28 of Bulletin No. 29. Pupils should master the writing of statements before being permitted to proceed to the next step of work.

45. **Wording of Problems:**

As far as possible, the situations in the problems of the bulletin are those with which pupils are familiar or which can readily be explained to them.

46. **Solution of Problems and Proof:**

Under each of the type problems, questions are asked to help pupils in their method of attacking the problem. In each case in percentage, emphasis is placed upon having the pupil first work out the meaning of the per cent in the problem. As part of the regular problem work, pupils are required to prove the answers to their problems whenever possible.

The problems of Case One on pages 52 through page 58 should receive special attention.

The tests on pages 60, 61 may be used as guides when planning tests on the problem work of Case One in percentage.

Formal Work Case Two in Percentage.

The second case in percentage is usually more difficult for pupils to master than either of the other cases. Experience has shown that pupils gain more power is attacking problems of this case when taught the method of unitary analysis rather than the method of the text. All explanations of difficulties of the formal work of Case II of percentage will be found on pages 62, through 72 of the bulletin. Care must be taken to see that pupils follow closely the work of the models.

The suggestions given on pages 69–70 are important.

The tests on pages 74–75 may be used as guides, when planning tests, on the formal work of Case II of percentage.

The Problem Work of Case Two in Percentage.

Problems similar to those on page 178 of text are very difficult for pupils to understand and also very impractical. Problems of this type have been entirely omitted from the work of the bulletin. Normal students are however required to be familiar with such problems.

Review of Cases One and Two.

The test of a pupil's knowledge of the first or second cases in percentage comes when the problems of these two cases are mixed. Such a review may be found on pages 90–95 of the bulletin.

Case Three Formal.

Pupils must be taught to change decimals to per cents before beginning the real work of Case Three. Each type of decimal is taught and applied in the various sections of pages 99 through 110 of the bulletin (Text 179-182). A test of the formal work of Case Three of percentage may be found on page 107.

Problem Work of Case Three.

The difficulties met in teaching the problems of Case III in percentage are explained and illustrated on pages 114 through 122 of the bulletin.

Review of Cases One, Two, Three.

The test of the pupil's knowledge of percentage comes when problems of the three cases are presented in a miscellaneous order. Such a review may be found on pages 184 through 191 of text or pages 122 through 126 of bulletin. Normal students are expected to be familiar with all of the above mentioned problems.

TEST ON FORMAL WORK OF PERCENTAGE.

The following miscellaneous review of the three cases of percentage may be used as a guide when it is necessary to test a pupil's knowledge of the three cases of percentage. At the beginning of a term or when a new pupil enters a class, such a test is useful. Pupils can be then drilled upon the points in which their knowledge is deficient.

1. Illustrate with a diagram and a problem the difference between $\frac{1}{4}$ of \$120 and $\frac{1}{4}\%$ of \$120.
2. Write each of the following expressions in two other ways:
   $101\frac{3}{4}\%$, $\frac{3}{5}\%$, 200%, 104%, 45%, $66\frac{2}{3}\%$, $\frac{3}{4}\%$, $1\frac{1}{2}\%$, $1\frac{1}{2}$, $\frac{1}{2}\%$, $1\frac{1}{4}\%$, $\frac{5}{6}\%$, 12%, 110%, $\frac{1}{10}$, $1\frac{1}{10}\%$, $\frac{2}{3}\%$, $133\frac{1}{3}\%$, $8\frac{1}{3}\%$, 250%, 6, 103%, 82%, $101\frac{5}{6}\%$, 85%, 111%, 2.
3. \$102 is $3\frac{1}{2}\%$ of what amount?
4. Find the entire cost of a book when $83\frac{1}{3}\%$ of the cost is \$2.50.
5. Write each of the following as per cents: $\frac{1}{4}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{2}{3}$, $\frac{3}{8}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{7}{10}$, $\frac{1}{3}$, $\frac{1}{8}$.
6. Change $\frac{19}{23}$ to a per cent.
7. What per cent of 2 is 1.75?
8. What per cent of \$810.20 is \$350?
9. Find in the shortest way—*a*. $166\frac{2}{3}\%$ of 270. *b*. $8\frac{1}{3}\%$ less than 180.
10. Express each of the following as per cents:
    a. 3.1, .0401, 1.1, 2.3, $2\frac{1}{2}$, .0075, .00125.
    b. .8906, $.24\frac{1}{2}$, .161, 1.001, 1.013, 2.5, .024.
    c. $1\frac{3}{4}$, .0075, .75, .010, 1.011, 1.111, 3.
    d. $2\frac{1}{5}$, .022, 1.4, 1.067, $2\frac{5}{9}$, 4.25, 3.2.
11. Find the number that is $17\frac{1}{2}\%$ less than 1,000.
12. \$850 is $1\frac{2}{3}\%$ of what amount?
13. Find 100% more than \$600.
14. Find $\frac{1}{4}\%$ less than 210.
15. What per cent of a mile is a rod?
16. Which is the largest amount, $\frac{83}{100}\%$ 17% or .96 of \$250?
17. What is the difference between 130% of \$210 and 30% more than \$210?
18. What is the difference between 5% less than \$115 and 95% of \$115?
19. Write a per cent of a quantity that is equal to less than the whole of a quantity. To more than the whole of the quantity?

20. Why is it incorrect to say that \$400 = 125%? Correct the statement.
21. Tell two ways of finding $33\frac{1}{3}\%$, 40% or 25% of a quantity.
22. Six out of every 100 is the same as how many per cent?
23. Is there any difference between $\frac{1}{8}$, $12\frac{1}{2}\%$ and $\frac{1}{8}\%$ of 64? Illustrate with work and a diagram.
24. What per cent of $\frac{7}{8}$ is $\frac{2}{3}$?
25. Tell without working the following problems in which cases the answers will be larger and in which cases smaller than the given quantity.
    a. Find 105% of 650. *b*. What is 4% less than 220? *c*. Find $\frac{1}{3}\%$ of 125. *d*. What is $2\frac{3}{4}\%$ more than 175? *e*. Find 200% of \$150.
26. What per cent of a quantity is always equal to a quantity?

TEST ON PROBLEM WORK OF PERCENTAGE.

(Text pages 171–190.)

The following test on the problem work of the three cases of percentage may be used as a guide when planning review work or as a test for new pupils who enter the class late in the term. Problems at the end marked with stars are the additional types with which Normal students are required to be familiar.

1. Mr. Hansen paid \$6,000 for a piece of property. He wishes to rent the property so that he may have a net income of 6% on his investment. His yearly expenses average \$140. Find the monthly rent Mr. Hansen must receive from his property in order to have a net income of 6%. (Difficult for pupils.)
2. Find the selling price of a horse sold at a loss of 15% which amounted to \$42.
3. Given the gain and the selling price, tell how to find the gain per cent and illustrate with a problem of your own.
4. At the close of the year 1913, there were 640 pupils in a certain school. How many more pupils must be enrolled during 1914 to have the enrollment at the end of the year 1914 25% greater than at the close of the year 1913?
5. When the gain and the gain per cent are given in a problem, how is the selling price obtained? Illustrate with a problem of your own.
6. If the increase per cent is required, what two things must be known? Illustrate with a problem of your own.
7. Mr. Saunders paid \$10,000 for a piece of property. The monthly rent from the property amounts to \$125. If the annual expenses average \$280, what is Mr. Saunders' net rate of income from the property?
8. State and solve a problem in which you are required to find the interest due on a sum of money borrowed for one year at 6% interest.
9. What per cent of $\frac{3}{4}$ of a yard is $\frac{2}{3}$ of a yard?

10. There were 285 words in a spelling test. George spelled 265 words correctly. Find his per cent of error.
11. There were 500 words in a spelling test. Albert missed 82 words. What was Albert's per cent of correct work?
12. The population of a certain town was 25,000 in 1900. In 1910, the population had increased 5,500. The population in 1910 was what per cent of the population in 1900? What was the per cent of increase?
13. By selling a horse for $30 more than it cost, Mr. Tompkins gained 20%. Find the cost and the selling price of the horse.
14. Mr. Hamilton bought tomatoes at the rate of three cans for $.25 and sold the tomatoes at the rate of two cans for $.25. Find the per cent of gain or loss.
15. There are 215 boys and 289 girls in a certain school. What per cent of the pupils are boys?
16. Mr. Hooper bought a lot for $3,000, upon which he built a house that cost $4,500. The yearly expenses of the property average $320. Find the monthly rent Mr. Hooper must receive from his property in order to have a net income of 6%.
17. Louise saves 20% of her yearly salary and spends $960. What is Louise's yearly income?
18. A public library had 7,500 books in circulation during the month of July. This number represented 75% of all the books in the library. Find the number of books not in circulation during the month of July.
19. Mr. Green receives an annual income of $1,200 from property which originally cost him $8,500. If the yearly expenses average $240, find Mr. Green's net rate of income.
20. There are 520 pupils in a certain school. 210 of the pupils are boys. What per cent of the school are girls?
21. *After losing 10% of its weight, a piece of ice weighed 20 pounds. What was its original weight?
22. *Mr. Harris drew $350 out of the bank. This amount was 25% more than the amount he had left in the bank. How much had he originally in the bank?
23. *Two men invested $10,000 in a grocery business. One man put in 20% more than the other man. Find the amount of capital invested by each of the partners.
24. *A ring marked at 20% above cost is sold at a discount of 5%. Find the gain or loss per cent.
25. *Goods marked 15% above cost are sold at a discount of 15%. Find the gain or loss per cent.
26. *If a merchant sells $\frac{3}{4}$ of a yard of silk for what $\frac{7}{8}$ of a yard cost him, find his gain or loss per cent.
27. *In 1911, there were 24,072 girls attending high school in California. This was 18% more than the number of boys attending. Find the number of pupils attending high schools in California during the year 1911.

28.*By selling a horse at a profit of $25, Mr. Lucas received 115% of the cost of the horse. What was the selling price of the horse?

29.*If four tablets are sold for the cost of five tablets, find the gain or loss per cent.

COMMISSION, INSURANCE, TAXES, DISCOUNT, INTEREST.

The material of pages 192 through 217 of the text may be supplemented by the explanations and drill work of Bulletin No. 30. The topics of commission, insurance, taxes, etc., are merely an application of the work of percentage to business transactions. Pupils usually fail to master such topics because they have no understanding of business situations or of the terms used in the problems. Normal students will be expected to be familiar with any of the problems of the text on pages 192 through 277 as well as all of the work in Bulletin No. 30. Students should strive to obtain as full a knowledge as possible of these business transactions.

Test on Commission, Insurance, Taxes, Interest.

The following miscellaneous review problems may be used as a guide when it is necessary to test a pupil's knowledge of commission, insurance, taxes, interest or discount. At the beginning of a term or when a new pupil enters a class, such a test is useful. Pupils can then be drilled upon the points in which their knowledge is deficient.

1. What must be given in a problem in order that you may find the rate of an agent's commission? Illustrate by stating and solving a problem of your own.
2. What is meant by the net proceeds of a sale? Illustrate by a problem of your own.
3. When cartage and other expenses are to be paid, is the agent's commission figured on the amount left after deducting expenses? Give a reason for your answer. Illustrate by stating and solving a problem of your own.
4. Name as many different occupations or situations as you can in which a man receives a commission as all or part of his pay for services rendered to others.
5. What must be known in a problem in order that you may find the amount of an agent's commission? Illustrate by stating and solving a problem of your own.
6. If the selling price and the net proceeds are given in a problem, tell how you may find the rate of the agent's commission. Illustrate by stating and solving a problem of your own.
7. What is the difference between the assessed value and the real value of a piece of property? Illustrate with a problem of your own.
8. What must you know in order to find the rate of taxation which will yield a certain amount of revenue? Illustrate by stating and solving a problem of your own.

9. If the amount of a man's tax bill and the rate of taxation are known, what amounts may be found? Illustrate by stating and solving problems of your own.
10. Is every property owner compelled to insure his property against loss by fire?
11. What must you know in order to find the amount of a man's tax bill? Illustrate by stating and solving a problem of your own.
12. In what three ways may a rate of taxation be expressed? Illustrate by stating and solving a problem of your own.
13. Name at least five things for which the money derived from taxes is used.
14. *a.* Name some of the items of expense of the national government.
    b. Name some of the sources of income of the national government.
    c. What is the difference between a specific duty and an ad valorem duty? Illustrate by stating and solving a problem of your own.
15. Is the rate of taxation the same for all taxpayers who live in a certain city, town or district?
16. Give some reasons why two men living in the same block may pay different rates of insurance.
17. Name at least five things which would raise the rate of a man's insurance.
18. Illustrate what is meant by the face of a policy. What is the premium?
19. How is it possible for an insurance company to make the premium on a three-year policy or a five-year policy cheaper than that of an annual policy renewed each year for the same length of time?
20. What must you have given in a problem in order to find the rate of insurance? Illustrate by stating and solving a problem of your own.
21. What must you have given in a problem in order to find the amount of the premium due on a three-year policy? Illustrate by stating and solving a problem of your own.
22. Name at least four important things which should appear in a policy.
23. If the amount of premium and the rate of insurance are given in a problem, what unknown amount may you find? Illustrate by stating and solving a problem of your own.
24. Mr. Wilkins' property valued at $8,000 was assessed for 60% of its value at the rate of $2.27 per $100. Find the amount of Mr. Wilkins' tax bill.
25. Mr. Ruskin received $45 for selling some fruit on a commission of 3%. Find the net proceeds of the sale.
26. What is the compound interest on $500 borrowed for $1\frac{1}{2}$ years at the rate of 6% if the interest is compounded semi-annually?

27. What was the total duty on six dozen table knives worth $4.50 per dozen if the specific duty is 16¢ on each knife and the ad valorem duty 15%?

28. A watch marked $45 was sold by a wholesale dealer to a retail merchant at a discount of 20%. The retail dealer sold the watch at its original marked price. Find the retailer's gain or loss per cent.

29. A hotel worth $50,000 was insured in one company for $12,000, in another company for $15,000 and in a third company for $10,000. If the hotel was damaged to the extent of $20,000, how much of the damage would each company pay?

    If the hotel were damaged to the extent of $40,000, what amount of the damage would each company pay?

30. What is the interest on $350 borrowed October 8, 1912, and returned with interest at 5% on November 6, 1913? (Use exact time.)

31. Mr. White's taxes amounted to $75. What was the actual value of his property if the tax rate was $2.04 per $100 and the property was assessed for 60% of its real value?

32. Mr. Roberts insured his house worth $6,000 for $\frac{3}{4}$ of its value for three years at $1.10 per $100. What was the amount of his premium?

33. A real estate agent sold two lots for $12,500. He returned $8,500 to the owner of the lots. What rate of commission did the agent charge?

MENSURATION.

The work in mensuration on pages 221 through 252 of text may be supplemented by the work of Bulletins Nos. 31 and 32. Normal students are expected to be familiar with the work of these bulletins as well as the work of above pages of the text.

Test Questions on Mensuration.

The following questions and problems are a test of the work of the text, pages 221 through 252 and of Bulletins Nos. 31 and 32. Normal students should be familiar with the answers to these questions.

1. Explain with the aid of a diagram why the area of an obtuse-angled triangle is $\frac{b \times a}{2}$

2. Write the formula for the area of a circle and for the circumference of a circle. State and solve a problem to illustrate each.

3. Illustrate with a drawing the difference, if any, between three square inches and a three-inch square.

4. State and solve a problem of your own in which you are required to find the area of the entire surface of a cylindrical log.

5. How do you know when to use the square measure and when to use the cubic measure in working with cylinders?

6. If the area of a rectangle is given in acres and the length of one side in rods, how is the perimeter of the rectangle found? Illustrate with a problem of your own.
7. How many acres are there in a section and one half of land?
8. Express in feet the length and the width of a square field the area of which is one acre.
9. Write the table of square measure.
10. What is the perimeter of a field one mile square? What is its area?
11. If the circumference of a circle is divided by $3\frac{1}{7}$ and that answer by 2, what is the result called?
12. If the area of the floor of a square room is one square rod, what is the length and width of the room expressed in feet?
13. With the aid of a diagram find the area of the largest circle which may be cut from a three-inch square.
14. Why is it that the longest oblique line of an obtuse-angled triangle is not the altitude of the triangle?
15. If the area of a right-angled triangle is 270 square rods, what is the area of the rectangle which has the same base and altitude as the triangle?
16. Write three formulas for the area of a triangle. State and solve a problem to illustrate the advantage of each formula.
17. Given the circumference of a circle, how may its radius be found? Illustrate by stating and solving a problem of your own.
18. Show by a diagram the relation between the diagonal of a rectangle and the longest side of a right triangle.
19. Draw a trapezoid. Assign dimensions to it and find its area in acres.
20. Is the altitude of a figure always drawn within the figure? Illustrate.
21. What is the name of the solid, all dimensions of which are equal?
22. Draw three different kinds of triangles. With dotted lines show the parallelogram of which each triangle is one half. Label the base and altitude of both the parallelograms and triangles. Assign dimensions to each triangle and find its area in acres.
23. What is the formula for the volume of a cylinder? Illustrate with a problem of your own.
24. Is the perimeter of a triangle, square, rectangle, or circle a line or a surface? What measure is used in finding the perimeter of a figure?
25. Illustrate what is meant by one line being perpendicular to another line.
26. Given the circumference of a circle, how may its area be found? Illustrate by stating and solving a problem of your own.
27. Illustrate with drawings the difference, if any, between two square inches, two inches, a two-inch cube, and a two-inch square.

28. What is the ratio of the diameter of a circle to its circumference? What is the ratio of the circumference of a circle to its diameter?
29. Draw as many different parallelograms as possible and label the base and altitude of each. Explain with the aid of a diagram why the formula for the area of a parallelogram with no right angles is the same as the formula for the area of a rectangle.
30. What is the perimeter and the area of a six-inch square?
31. Tell for what each of the following numbers stands in the square table: 640, $30\frac{1}{4}$, 9, 144, 160, $208\frac{3}{4}$.
32. Is it entirely correct to say that a triangle is one half of a parallelogram? If not, make a correct statement.
33. Write the table of Linear Measure.
34. Is the circumference of a circle a surface or a line?
35. With a problem, illustrate the difference between finding the surface and the volume of a cylinder.
36. Draw a circle. Shade its surface and label its center, radius and circumference.
37. Given the circumference of the base of a cylinder, how is the area of the base of the cylinder found? Illustrate by stating and solving a problem of your own.
38. How do you find the area of the convex surface of a cylinder when the circumference of the base of the cylinder and the altitude of the cylinder are given? Illustrate by stating and solving a problem of your own.
39. A tree was broken by the wind in such a way that the top of the tree struck the ground at a distance of 45 feet from the foot of the tree. If the broken part of the tree was 65 feet long, find the length of the part left standing.
40. A lot is 125′ x 340′. How much shorter is it to walk diagonally through the lot than around the two adjoining sides of the lot?
41. Which field has the greater perimeter and how much greater, a rectangular field 210 feet by 350 feet or a square field of equal area?
42. The area of a square field is 8 acres. What is the perimeter of the field expressed in feet?
43. Which has the greater area, a rectangular table top 4′ 3″ x 3′ or a circular table top which has a diameter of 4′ 3″? How much larger or smaller is the rectangular table top than the circular table top?
44. The perimeter of a square field is 320 rods. At $65 an acre, what is the value of a triangular field which has the same base and altitude as the square?
45. Mr. Hamilton owns a farm one eighth of a mile square. Mr. White's farm has an area of one eighth of a square mile. What is the value of each farm at $125 an acre?
46. If a horse is tied to the bottom of a stake by a rope 20 feet long, what is the area of the surface over which the horse may graze?

47. How many feet of picture molding are needed for a room the length of which is 18 feet and the area of the floor 378 square feet?

48. Ethel is embroidering a round centerpiece for her mother for Christmas. If the greatest width of the centerpiece is 36 inches, how many inches of the outer edge of the centerpiece will Ethel have to embroider each day in order to finish the outer edge of the centerpiece in 8 days? How many square feet of the surface of a table will be covered by the centerpiece when finished?

49. A rectangular field 114 rods long has an area of $1\frac{1}{2}$ acres. If land is worth $250 an acre, what is the value of a triangular field which has the same base and altitude as the rectangle?

50. The length of the upper base of a trapezoid is 12 feet and the lower base is 8 feet. If the perpendicular distance between the sides is 4 feet 6 inches, find the number of square feet on the surface of the trapezoid.

51. How many revolutions will a 28-inch bicycle wheel make in traveling a mile?

52. How many cubic feet of marble are there in a column of marble 20 feet long and 22 inches in diameter?

53. How many rods of fence are needed to enclose a square field the area of which is 8 acres?

54. At $\$.12\frac{1}{2}$ a square foot, find the cost of laying a three-foot cement walk around a circular flower bed 30 feet in circumference.

55. What is the diameter of a circle which has an area equal to that of a seven-foot square?

56. The hypotenuse of a right triangle is 70 feet. The altitude of the triangle is 30 feet. What is the perimeter of the triangle?

57. What is the length of the longest stick which may be placed within a box cubical in shape, each edge of which measures 2 feet?

58. Express in gallons the capacity of a covered cylindrical tank which is 20 feet long and 3 feet in diameter.

59. What is the area of the entire outer surface of the above tank? (Include cover.)

60. Tell some method of proving that the ratio of the circumference of a circle to its diameter is $3\frac{1}{7}$.

61. A cistern filled with water is 6 feet in diameter and 40 feet deep. How many gallons of water does it contain?

62. If there are $31\frac{1}{2}$ gallons in a barrel, how many barrels of water may be filled from this cistern?

63. If the bottom and sides of the interior of the tank are cemented, find the cost at $.12 a square foot.

64. A marble column is 12 feet high and 38 inches in circumference. What is the area of the convex surface of the column?

65. What is its volume expressed in cubic yards?

66. What is the length of the diagonal of a square, each side of which is 10 feet?

CLASSIFICATION OF PROBLEMS OF PAGES 18–160 OF ADVANCED TEXT.

By grouping under their proper headings all problems of like types which are scattered throughout the various pages of the text, there will be found to be in most instances, sufficient problems to teach a given type.

Pupils who are accurate and understand their work quickly should not be required to work as many problems of a given type as pupils who are slow and inaccurate. The problems under each type have been divided into three groups—*a*, problems not marked with stars; *b*, problems marked with one star; *c*, problems marked with two stars. Pupils who work the non-starred problems correctly should be permitted to skip the single and double starred exercises.

Teach pupils to write a statement telling what is given and what is required to be found in each of the following problems:

Addition and Subtraction of Integers and Decimals.

Page 19—5–6; *7–8; **9–10–11. Page 23—5–7–9; *6–8. Page 24—2–3–7; *4–5. Page 25—6; *5–7. Page 28—1–2–3–4–5–6–7. Page 30—4–5–6. Page 73—5–6–7. Page 36—1 through 8.

Multiplication and Division of Integers and Decimals.

Page 35—Sec. 37, 2–3–4. Page 41—Sec. 45, 3. *Page 159—7–8–9–10. Page 130—12. Page 50—Sec. 58, 2–3; *4–5; **6. Page 50—Sec. 59, 1–3; *2–4. Page 53—3–5–8; *4–6–7–9. Page 56—Sec. 67, 1–2; *3–4–5. Page 56—Sec. 68, 1–4–5; *2–3. Page 67—Sec. 86, 1–2–4–7. Page 72—9. *Page 67—3–5–6–8. **Page 129—10–11. Page 159—1–2–4–12–13; *3–5–14. Page 56—Secs. 67–68 (entire). Page 164—6–7–8–9–10–11–12–13–14–15–16–23–24.

Addition and Subtraction of Fractions.

Page 98—4–8–9–10. Page 130—22–23. Page 110—2–3. *Primary Text page 188—1–2. *Page 190—5–6–7–8. *Page 191—5–6–7. *Page 192—5–6. **Page 203—1–2–4–5–6. Page 204—1–2–3.

Multiplication and Division of Fractions.

Page 118—4. Page 144—Sec. 187, 4–5. Page 130—25. Page 127—6–7–8–10–11. Page 127—Sec. 171, 2. Page 128—8–9–10–11. Page 144—Sec. 187, 3–6. Page 118—Sec. 160, 2–3–5.

Given Whole to Find Part.

Page 57—2–3–4–6–9–11; *5–7–10–12; **8–13–14. Page 58—1–4; *2–3; **5–6. Page 129—6–7. Page 144—Sec. 187, 1.

Given Part to Find Whole.

Page 59—Sec. 72, No. 1 through 12. Page 139—2–4–5–7–9–10. *Page 129—8–9. Page 139—3–6–8–11–12–14–15–17–23; **13–16–18–19–20–21–22–24–25–26–27. Page 141—Sec. 183, entire. Page 143—3–5–6–7–8–10; *1–2–4–9–11. Page 144—Sec. 187, 2.

Profit and Loss.

Page 142—14–15–16. Page 152—Sec. 199.

Bills and Receipts.

Pages 42–43. Page 61—1–2. Bull. No. 29—Pages 41–42–43–97–98. See Test Sec. 39 of this book.

Average and Ratio.

Page 41—Sec. 45. 1. Page 118—Sec. 160. 6. Page 54—55–145–146. Bull. 29—Pages 73–74.

Area of Rectangles.

The following problems of the text may also be used as additional drill work while teaching pages 80 to 84:

Page 130—No. 15–16–17–18.

Page 128—No. 7–15–17.

Page 129—No. 1–2–3–4–5.

Page 165—No. 24.

See Test Sec. 42 of this book.

Cubic Measure.

Pages 84–85–86.

Page 236—6–7.

SAN FRANCISCO STATE NORMAL SCHOOL PUBLICATIONS.

The Teachers' Manuals.

Some years ago the San Francisco State Normal School undertook the publication of courses of study for teaching the various subjects of the curriculum of the elementary school. These courses were prepared by members of the faculty and were the outgrowths of daily experience in directing and supervising the teaching by student teachers in the Normal Elementary School. The original purpose of these publications was to furnish to these student teachers directions for teaching each of the subjects. An essential necessity in their construction was that they should be very explicit, specific and practicable in use. Gradually there grew a demand for them by teachers in the public schools, and the Normal School began to print larger editions in order to fill this new need. The demand from the public school sources has now grown to such proportions that one chief service of the institution is that of its publications.

Pupils' Exercise Books.

Up to 1912 the publications had been confined to courses of study for the assistance of teachers. During 1912 the publication of pupils' exercise books, accompanying the teachers' bulletins, was commenced. In one type of these exercise books the pupils write directly in printed lessons. This device saves a large amount of labor and time of the teacher in copying upon the board and in oral instructions. Further, it saves the pupils' time in copying from the board. But pupils can make progress two or three times faster than by the usual method, and the work is done much more effectively and without the sense of drudgery either to pupil or teacher. The exercise books are printed upon paper that will take ink. They cost little or no more than the common blank books of the same quality of paper.

Monographs.

There is now commenced a series of monographs of a practical nature, aimed to assist or suggest further development of a greater efficiency of school instruction.

Three Series.

There have been three series of publications in time—one issued prior to the great fire of 1906, of which no numbers now remain; a series begun in 1907 and continued to 1914, and, finally, the Pupil's Self-Instruction Series, begun in 1914. The latter two will be found listed below.

System of Publication.

The expense of these publications is borne chiefly by a revolving fund obtained by their sale. They are printed in the State Printing Office and sold practically at manufacturing cost. They are issued merely upon the authority of the individual authors and the editor of the series, and do not represent a general or necessarily permanent policy of the school, nor a consensus of its faculty or trustees.

How to Order.

All orders must be accompanied by school district warrant check, money order or stamps. We cannot fill orders which require keeping of accounts. As most of the purchases of bulletins and pupils' exercise books are now made by the school districts, teachers who send orders should be careful to secure the signature of trustees to warrants in payment for orders, so that delays may be avoided. Be careful, also, in filling out orders that the bulletins are listed and are not out of print. We cannot exchange publications once purchased unless error has been made and the request is made within three days. Kindly avoid, so far as possible, conditions which require special correspondence in the business department.

MONOGRAPHS.

Monograph A. A remedy for Lock-Step Schooling; a preliminary report upon the weakness and impossibilities of the class system of instruction, and progress to date in substituting therefor an individual system of teaching. By Frederic Burk. Issued free upon application.

Monograph B. Outline courses in general information and general intelligence. This monograph undertakes to map out the beginning of a reorganization of the high school course of study, not only for the better preparation of those intending to become teachers, but also in the general cause of wider preparation of all students in industrial, civic and social intelligence. To students intending to enter the San Francisco State Normal School

the monograph will be sent free. Section I printed as separate pamphlet; American History and Civics, by P. F. Valentine. Section II, pamphlet for General History, Science, and Literature, Arithmetic, Geography, and Music. Section III, Spelling, Language, and Grammar. Price—the three sections will be sent for 25 cents, postpaid; in lots of 25 or more, expressage or freight paid by purchaser, 7½ cents per section.

Monograph C. (In preparation.) This will be a sequel to Monograph A, giving the data of results of one year's experience in operating an individual system of instruction in the Normal Elementary Department. It will show the records of about 500 pupils, their variations in rates of progress, etc.

Monograph D. Critical Difficulties in the Teaching of Arithmetic. For teachers, and for students preparing for admission to the Normal School. By Mary A. Ward. Price 15 cents, 3 cents added for postage.

PUPIL'S SELF-INSTRUCTION SERIES.

(Adapted to an Individual Method of Teaching.)

The occasion and general plan for this series is set forth in Monograph A. We are at this date (August, 1914) beginning the publication of a series of pupils' exercise books and teachers' manuals adapted to use under the individual system of instruction. They, of course, may also be used under the class system and will assist in teaching by the state series texts. Their plan of construction embodies the features outlined in Monograph A—the "elastic" lesson, by which the number of exercises to secure comprehension or accuracy varies according to individual need; the adaptation to simplicity of language; the lesson directions whereby the pupil can make his own rate of progress and, to a large extent, independently of prescribed lessons or help from the teacher; the cumulative reviews by which all principles once learned are carried forward automatically.

This series will be found invaluable, especially for the rural school where pupils must depend largely upon their own resources. The teachers' manuals will give full directions for operation of the system.

Prices.

Except where specially stated, the price of the numbers of the Self-Instruction Series will be as follows:

At the Normal School, 10 cents each;

By mail, 12 cents each, postpaid;

By express or freight, $7.50 per hundred, transportation charges paid by buyer.

Arithmetic. By Frederic Burk and Mary A. Ward.

No. 20—Teachers' Manual to accompany pupils' books, Nos. 21, 22, and 23, giving directions, answers to examples and supplementary examples. Price—25 cents each, postpaid.

No. 21—Pupils' Exercise Tablet in addition and subtraction.

No. 22—Pupils' Exercise Tablet in multiplication and short division.

No. 23—Pupils' Exercise Tablet in compound multiplication and long division.

No. 29—Problems in Percentage. Double number. Price—20 cents each at Normal School; 24 cents by mail; $15.00 per hundred by express.

No. 30—Applications of Percentage.

No. 31—Problems in Mensuration, Part I.

No. 32—Problems in Mensuration, Part II.

Language. By A. S. Boulware and Ethel G. Smith.

No. 42—Pupils' Exercise Tablet in Language, Part I (for fifth grades).

No. 43—Pupils' Exercise Tablet in Language, Part II.

No. 44—Pupils' Exercise Tablet in Language, Part III.

Grammar. By Ethel G. Smith and Frederic Burk.

No. 51—Part I.

No. 52—Part II.

No. 53—Part III.

No. 54—Part IV.

No. 55—Part V. (In preparation.)

No. 56—Part VI. (In preparation.)

History. By P. F. Valentine.

No. 80—Pupils' Exercise Book, Part I, to accompany advanced state text. (Columbus through Jefferson.)

No. 81—Pupils' Exercise Book, Part II, to accompany advanced state text. (Madison through Civil War.)

No. 82—Pupils' Exercise Book, Part III, to accompany advanced state text. (Civil War to present.)

No. 83—Difficulties of history texts simply explained. (Democracy, the Constitution, Centralized Government, Religious Toleration, Monroe Doctrine, Spoils System, Civil Service Reform, the Tariff, etc.)

Geography. By F. W. Hoffmann.

Bulletin No. 18, Teachers' Manual, with two pupils' exercise books in Map Geography, is partly constructed upon the individual plan. It is already published and may be obtained upon application. (See next list.)

IN PREPARATION.

There are in preparation, for publication during the year, the following:

Arithmetic. Two tablets in formal arithmetic for fractions and decimals; one exercise book in arithmetic problems in integers; one exercise book in denominate numbers; one exercise book in application of percentage.

Language. Two exercise tablets for third and fourth grades, respectively.

Phonics. A series of exercise books.

Writing. A series of exercise books.

Drawing. A series of exercise books.

Music. A series of exercise books in formal note work.

TEACHERS' MANUALS AND PUPILS' EXERCISE BOOKS.

(Series Published 1907 to 1914.)

(Out of print: Nos. 1, 2, 3, 5, 6, 7, 8 and 13. The materials of these have largely been absorbed in revised editions represented by the later numbers.)

No. 4—A Course of Study in Map Geography; paper bound, 52 pages. By Allison Ware. Price—by mail, postpaid, 30 cents.

Outline Maps—In connection with Bulletin No. 4, the school publishes a series of nine outline maps from which pupils may trace outlines for use in location. These maps are 9 by 12 inches in size. They represent the following areas: North America, South America, Europe, Asia, Africa, Australia, United States, California, and the hemispheres. Price—by mail, postpaid, for set of nine, 15 cents. (See also Bulletin No. 18.)

No. 9—A Course of Study in Language; 174 pages. By Effie Belle McFadden. Price—by mail, postpaid, 30 cents.

No. 10—A Course of Study and Teachers' Handbook in the Common Literature of Life; 207 pages, paper bound. By Allison Ware. Price—postpaid, 40 cents.

No. 11—A Course of Study in Formal Arithmetic and Teachers' Handbook. By David Rhys Jones. This bulletin is published in various parts as follows:

Part I. Teachers' Handbook and exercises for integers; 109 pages, paper bound. Price—by mail, postpaid, 30 cents.

Part II. Teachers' Handbook and exercises for common fractions, decimals, percentage, denominate numbers and mensuration; 84 pages, paper bound. Price—by mail, postpaid, 30 cents.

The Pupils' Exercise Books, Nos. 1, 2, and 4, accompanying the Handbooks, are out of print; a limited stock of No. 3 (fractions and decimals) is still on hand. Price—10 cents by mail; the pupils' work, however, is included in the Teachers' Handbook. Individual Series No. 21 takes the place of No. 1; No. 22 and No. 23 take the place of No. 2.

No. 12—Review Courses of American History by means of Composition Topics, and Teachers' Handbook to the use of the California State Series Texts. By P. F. Valentine.

Part I. Teachers' edition containing introduction and directions for use of the composition method, the pupils' topics of the primary text, the pupils' topics of the advanced text, a paragraph directory to the text, and a cumulative fact review of the advanced text; 73 pages. Price—postpaid, 25 cents.

Part II. Pupils' edition containing the composition outlines which follow the state primary text in history; 10 pages. Price—postpaid, 5 cents; in lots of 25 or more, freight or expressage paid by purchaser, 4 cents per copy.

Part III. Pupils' edition containing the composition outlines which follow the state series advanced text in history, the paragraph directory to the text, and the cumulative fact review for the same; 48 pages. Price—postpaid, 10 cents; in lots of 25 or more, freight or expressage paid by purchaser, 8 cents per copy.

No. 13—A Course of Study in Applied Problems in Arithmetic, for Supplementary Use. By Mary A. Ward.

Out of print—order No. 29, Self-Instruction Series.

No. 14—A Course of Study in the Teaching of Composition, Language and Spelling; paper bound. By Effie B. McFadden, assisted by Ethel G. Smith. Teachers' edition for first three years. Price—postpaid, 25 cents. Nos. 1 and 2 of the pupils' exercise books heretofore accompanying this handbook are out of print; they are replaced by Nos. 40 and 41, Self-Instruction Series. A small stock of Nos. 3 and 4 still remains. Price—10 cents, postpaid.

No. 15—A Simplified Course of Study in the Teaching of Grammar; teachers' edition, paper bound, 187 pages. By Frederic Burk, Effie B. McFadden and Irving Brazier. Price —postpaid, 40 cents. The pupils' exercise books accompanying this handbook are all out of print and are being replaced by Nos. 51, 52, 53, and 54 of the Self-Instruction Series.

No. 16—A Course of Study in Phonics. By Corrine H. Johnstone and Frederic Burk. Teachers' Edition, Part I, 90 pages. Price—postpaid, 20 cents.

Pupils' Phonic Exercise Book No. 1 (containing exercises reprinted from teachers' edition). Price—postpaid, 10 cents; in lots of 25 or more, freight or expressage paid by purchaser, 7½ cents.

No. 17—A Composition Course in American Government and Pupils' Handbook to the State Series Text (Dunn's Community and Citizen) with Supplement containing revised or additional paragraphs upon conservation, some California laws, Interstate Commerce Commission, California's compulsory education, direct election of United States senators, direct primary, township and county, the commission form of city government, the initiative, referendum and recall, the cabinet; paper bound, about 40 pages. By P. F. Valentine. Price—postpaid, 15 cents; in lots of 25 or more, expressage or freight paid by purchaser, 10 cents per copy.

No. 18—A Course of Study in Map Geography; can be used in grades as low as fourth. By F. W. Hoffman.

Teacl ooks. Price—10 cents, postpaid.

Pupil pupils write directly. Price—12 c

Pupils dual instruction, with maps and stpaid; in lots of 25 or more, freig r both exercise books.

Outlin given for mounting); No. 1, wor Price—10 cents each, postpaid.

Pupils ator and California). Price—10 c